CHAPTER

1

SETS

Animation 1.1: Sets-math
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Student Learning Outcomes

After studying this unit, students will be able to:

- Express a set in:
  - descriptive form,
  - set builder form,
  - tabular form.
- Define union, intersection and difference of two sets
- Find:
  - union of two or more sets,
  - intersection of two or more sets,
  - difference of two sets
- Define and identify disjoint and overlapping sets
- Define a universal set and compliment of a set
- Verify different properties involving union of sets, intersection of sets, difference of sets and compliment of a set, e.g $A \cap A' = \phi$.
- Represent sets through Venn diagram.
- Perform operation of union, intersection, difference and complement on two sets A and B, when:
  - A is subset of B,
  - B is subset of A,
  - A and B are disjoint sets,
  - A and B are overlapping sets, through Venn diagram.

1.1 Introduction

In our daily life, we use the word set only for some particular collections such as water set, tea set, dinner set, sofa set, a set of books, a set of colours and so on.

But in mathematics, the word set has broader meanings than those in our daily life because it provides us a way to integrate the different branches of mathematics.

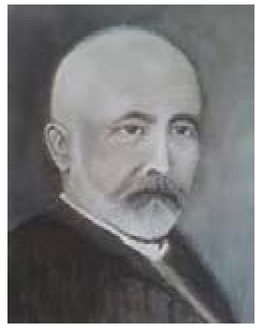
It also helps to solve many mathematical problems of both simple and complex nature. In short, it plays a pivotal role in the advanced study of the mathematics in the modern age. Look at the following examples of a set.

A = The set of counting numbers.
B = The set of Pakistani Provinces.
C = The set of geometrical instruments

Recall

A set cannot consist of elements like moral values, concepts, evils or virtues etc.

" A set is a collection of well defined objects/numbers. The objects/numbers in any set are called its members or elements"



"Set theory" is a branch of mathematics that studies sets. It is the creation of George Cantor who was born in Russia on March 03, 1845. In 1873, he published an article which makes the birth of set theory. George Cantor died in Germany on January 06, 1918

1.1.1 Expressing a Set

There are three ways to express a set.

1. Descriptive form 2. Tabular form 3. Set builder form

- **Descriptive form**

If a set is described with the help of a statement, it is called as descriptive form of a set.

For Example:

N = set of natural numbers
Z = set of integers
P = set of prime numbers
W = set of whole numbers
S = set of solar months start with letter "J"

Do you Know
The sets of natural numbers, whole numbers, integers, even numbers and odd numbers are denoted by the English letters N, W, Z, E and O respectively.

- **Tabular Form**

If we list all elements of a set within the braces { } and separate each element by using a comma "**,**" it is called the tabular or roster form.

For Example:

A = {*a, e, i, o, u*} C = {3, 6, 9, ... ,99}
M = {*football, hockey , cricket*} N = {1, 2, 3, 4, ...}
W = {0, 1, 2, 3, ...} X = {*a, b, c, ..., z*}

- **Set Builder Form**

If a set is described by using a common property of all its elements, it is called as set builder form. A set can also be expressed in set builder form. For example , "E is a set of even number" in the descriptive form, where E = {0, ±2, ±4, ±6,.....} is the tabular form of the same set. This set in set builder form can be written as;

E = {x | x is an even number}

and we can read it as, E is a set of elements x, such that x is an even number.

A = {x | x is a solar month of a year}
B = {x | $x \in$ N $\wedge$ 1 < x < 5}
C = {x | $x \in$ W $\wedge$ x $\leq$ 4}

Some Important Symbols	
| such that	$\in$ belongs to
$\wedge$ and	$\vee$ or
$\geq$ greater than or equal to	
$\leq$ less than or equal to	

EXERCISE 1.1

1. Write the following sets in descriptive form.

(i) A = {*a, e, i, o, u*} (ii) B = {3, 6, 9, 12, ...}
(iii) C = {*s, p, r , i, n, g*} (iv) D = {*a, b, c, ... ,z*}
(v) E = {6, 7, 8, 9, 10} (vi) F = {0, ±1, ±2}
(vii) G = {x | $x \in$ N $\wedge$ x < 3} (viii) H = {x | $x \in$ N $\wedge$ x > 99}

2. Write the following sets in tabular form.

(i) A = Letters of the word "hockey"
(ii) B = Two colours in the rainbow
(iii) C = Numbers less than 18 divisible by 3
(iv) D = Multiples of 5 less than 30
(v) E = {x | $x \in$ W $\wedge$ x > 5}
(vi) F = {x | $x \in$ Z $\wedge$ – 7 < x < –1}

3. Write the following sets in the set builder form.

(i) A = {1,2,3,4,5}
(ii) B = {2,3,5,7}
(iii) N = set of natural numbers
(iv) W = set of whole number
(v) Z = set of all integers
(vi) L = {5, 10, 15,20,...}
(vii) E = set of even numbers between 1 and 10
(viii) O = set of odd numbers greater than 15
(ix) C = set of planets in the solar system
(x) S = set of colours in the rainbow

1.2 Operations on Sets

1.2.1 Union, Intersection and Difference of Two Sets

- **Union of Two Sets**

The union of two sets A and B is a set consisting of all the elements which are in set A or set B or in both. The union of two sets is denoted by A ∪ B and read as "A union B"

Example 1: If A = {*a, e, i, o*} and B = {*a, b, c*}, then find A ∪ B

Solution:

A = {*a, e, i, o*}, B = {*a, b, c*}
A ∪ B = {*a, e, i, o*} ∪ {*a, b, c*}
= {*a, e, i, o, a, b, c*}

Example 2: If M = {1, 2, 3, 4, 5} and N = {1, 3, 5, 7}, then find M ∪ N

Solution:

M = {1, 2, 3, 4, 5}, N = {1, 3, 5, 7}
M ∪ N = {1, 2, 3, 4, 5} ∪ {1, 3, 5, 7}
= {1, 2, 3, 4, 5, 7}

- **Intersection of Two Sets**

The intersection of two sets A and B is a set consisting of all the common elements of the sets A and B. The intersection of two sets A and B is denoted by A ∩ B and read as "A intersection B"

Example 3: If A = {*a, e, i, o, u*} and B= {*a, b, c, d, e*}, then find A ∩ B

Solution:

A = {*a, e. i, o, u*}, B = {*a, b, c, d, e*}
A ∩ B = {*a, e, i, o, u*} ∩ {*a, b, c, d, e*}
= {*a, e*}

Animation 1.2: Intersection of two sets
Source & Credit: elearn.punjab

Example 4: If X = {1, 2, 3, 4} and Y = {2, 4, 6, 8}, then find X ∩ Y

Solution:

X = {1, 2, 3, 4}, Y= {2, 4, 6, 8}
X ∩ Y = {1, 2, 3, 4} ∩ {2, 4, 6, 8}
= {2, 4}

- **Difference of Two Sets**

Consider A and B are two any sets, then A difference B is the set of all those elements of set A which are not the elements of set B. It is written as A – B or A \ B. Similarly, B difference A is the set of all those elements of set B which are not the elements of set A. It is written as B – A or B \ A.

Example 5: If A = {1, 3, 6} and B = {1, 2, 3, 4, 5}, then find:
(i) A – B (ii) B – A

Solution:

A = {1, 3, 6}, B = {1, 2, 3, 4, 5}
(i) A – B = {1, 3, 6} – {1, 2, 3, 4, 5}
= {6}
(ii) B – A = {1, 2, 3, 4, 5} – {1, 3, 6}
= {2, 4, 5}

1.2.2 Union and Intersection of Two or More Sets

We have learnt the method for finding the union and intersection of two sets. Now we try to find the union and intersection of three sets.

- **Union of three sets**

Following are the steps to find the union of three sets

Step 1: Find the union of any two sets.
Step 2: Find the union of remaining 3rd set and the set that we get as the result of the first step
For three sets A, B and C their union can be taken in any of the following ways.

(i) A ∪ (B ∪ C) **(ii)** (A ∪ B) ∪ C

It will be easier for us to understand the above method with examples. Look at the given examples.

Example 6: Find A ∪ (B ∪ C) where A = {1, 2, 3, 4}, B = {3, 4, 5, 6, 7, 8} and C = {6, 7, 8, 9, 10}.

Solution:

A ∪ (B ∪ C) = {1, 2, 3, 4} ∪ {3, 4, 5, 6, 7, 8} ∪ {6, 7, 8, 9, 10}
= {1, 2, 3, 4} ∪ {3, 4, 5, 6, 7, 8, 9, 10}
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Example 7: If A = {1, 3, 7}, B = {3, 4, 5} and C = {1, 2, 3, 6}

Solution:

(A ∪ B) ∪ C = {1, 3, 7} ∪ {3, 4, 5}) ∪ {1, 2, 3, 6}
= {1, 3, 4, 5, 7} ∪ {1, 2, 3, 6}
= {1, 2, 3, 4, 5, 6, 7}

- **Intersection of Three Sets**

For finding the intersection of three sets, first we find the intersection of any two sets of them and then the intersection of the 3rd set with the resultant set already found.

(i) A ∩ (B ∩ C) **(ii)** (A ∩ B) ∩ C

Example 8: Find A ∩ (B ∩ C) where A = {*a, b, c, d*}, B = {*c, d, e*} and C = {*c, e, f, g*}

Solution:

A ∩ (B ∩ C) = {*a, b, c, d*} ∩ ({*c, d, e*} ∩ {*c, e, f, g*})
= {*a, b, c, d*} ∩ {*c, e*}
= {*c*}

Example 9: If A = {1, 2, 3, 4}, B= {2, 3, 4, 5} and C = {1, 2}, then find (A ∩ B) ∩ C

Solution:

(A ∩ B) ∩ C = ({1, 2, 3, 4} ∩ {2, 3, 4, 5}) ∩ {1, 2}

= {2, 3, 4} ∩ {1, 2} = {2}

EXERCISE 1.2

1. Find the union of the following sets.

(i) A = {1,3,5}, B = {1,2,3,4}
(ii) S = {*a, b, c*}, T = {*c, d, e*}
(iii) X = {2,4,6,8,10}, Y = {1,5,10}
(iv) C = {*i, o, u*}, D = {*a, e, o*}, E = {*i, e, u*}
(v) L = {3, 6, 9, 12}, M = {6, 12, 18, 24,}, N = { 4, 8, 12, 16}

2. Find the intersection of the following sets.

(i) P = {0, 1, 2, 3}, Q = {–3, –2, –1, 0}
(ii) M = {1, 2, ... , 10}, N = {1, 3, 5, 7, 9}
(iii) A = {3, 6, 9, 12, 15}, B = {5, 10, 15, 20}
(iv) U = {–1, –2 , –3}, V = {1, 2, 3}, W = {0, ±1, ±2}
(v) X = {*a, l, m*}, Y = {*i, s, l, a, m*}, Z = {*l, i, o, n*}

3. If N = set of Natural numbers and W = set of Whole numbers, then find N U W and N ∩ W

4. If P = set of Prime numbers and C = set of Composite numbers, then find P U C and P ∩ C

5. If A = {a, c, d, f}, B = {b, c, f, g} and C = {c, f, g, h}, then find
(i) A U (B U C) (ii) A ∩ (B ∩ C)

6. If X = {1, 2, 3, ..., 10}, Y = {2, 4, 6, 8, 12} and Z = {2, 3, 5, 7, 11}, then find:
(i) X U (Y U Z) (ii) X ∩ (Y ∩ Z)

7. If R = {0, 1, 2, 3}, S = [0, 2, 4) and T = {1, 2, 3, 4}, then find:
(i) R \ S (ii) T \ S (iii) R \ T (iv) S \ R

1.2.3 Disjoint and Overlapping Sets

- **Disjoint Sets**

Two sets A and B are said to be disjoint sets, if there is no common element between them. In other words there intersection is an empty set, i.e. A ∩ B = ϕ. For example, A = {1, 2, 3}, B = {4, 5, 6} are disjoint sets because there is no common element in set A and B.

- **Overlapping Sets**

Two sets A and B are called overlapping sets, if there is at least one element common between them but none of them is a subset of the other. In other words, their intersection is non-empty set. For example, A = {0, 5, 10} and B= {1, 3, 5, 7} are overlapping sets because 5 is a common element in sets A and B and non is subset of the other.

1.2.4 Universal Set and Complement of a Set

- **Universal Set**

A set which contains all the possible elements of the sets under consideration is called the universal set. For example, the universal set of the counting numbers means a set that contains all possible numbers that we can use for counting. To represent such a set we use the symbol U and read it as "Universal set" i.e.

The universal set of counting numbers: U = {1, 2, 3, 4, ...}

- **Complement of a Set**

Consider a set B whose universal set is U, then the difference set U \ B or U - B is called the complement of a set B, which is denoted by B′ or B^c and read as "B complement". So, we can define the complement of a set B as: "B complement is a set which contains all those elements of universal set which are not the elements of set B, i.e. B′ = U \ B.

Example 1: If U = {1, 2, 3, ..., 10} and B = {1, 3, 7, 9}, then find B′.

Solution:

U = {1, 2, 3, ...,10}, B = {1, 3, 7, 9}
B′ = U – B
= {1, 2, 3, ... , 10} – {1, 3, 7, 9}
= {2, 4, 5, 6, 8, 10}

EXERCISE 1.3

1. Look at each pair of sets to separate the disjoint and overlapping sets.

(i)	A =	{*a, b, c, d, e*},	B =	{*d, e, f, g, h*}
(ii)	L =	{2, 4, 6, 8, 10},	M =	{3, 6, 9, 12}
(iii)	P =	Set of Prime numbers,	C =	Set of Composite numbers
(iv)	E =	Set of Even numbers,	O =	Set of Odd numbers

2. If U = {1, 2, 3, ..., 10}, A = {1, 2, 3, 4, 5}, B = {1, 3, 5, 7, 9}, C = {2, 4, 6, 8, 10} and D = 3, 4, 5, 6, 7}, then find:

(i) A′ (ii) B′ (iii) C′ (iv) D′

3. If U = {*a, b, c,...., i* }, X = {*a, c, e, g, i*}, Y = {*a, e, i*}, and Z = {*a, g, h*}, then find:

(i) X′ (ii) Y′ (iii) Z′ (iv) U′

4. If U= {1, 2, 3, ..., 20}, A= {1, 3, 5, ... ,19} and B = {2, 4, 6, ... ,20}, then prove that:

(i) B′ = A (ii) A′ = B (iii) A \ B = A (iv) B \ A = B

5. If U = set of integers and W = set of whole numbers, then find the complement of set W.

6. If U = set of natural numbers and P = set of prime numbers, then find the complement of set P.

1.2.5 Properties involving Operations on Sets

We have learnt the four operations of sets, i.e. union, intersection, difference and complement. Now we discuss their properties.

- **Properties involving Union of Sets**

- **Commutative property**

If A, B are any two sets, then "**A ∪ B = B ∪ A**" is called the commutative property of union of two sets.

Example 1: If A= {1, 2, 3} and B = {2, 4, 6}, then verify that:
A ∪ B = B ∪ A.

Solution:

A U B = {1, 2, 3} ∪ {2, 4, 6}
= {1, 2, 3, 4, 6}
B U A = {2, 4, 6} ∪ {1, 2, 3}
= {1, 2, 3, 4, 6}

From the above, it is verified that:

A ∪ B = B ∪ A

- **Associative Property**

If A, B and C are any three sets, then "A ∪ (B ∪ C) = (A ∪ B) ∪ C" is called the associative property of union of three sets.

Example 2: If A = {1, 2, 3, 4, 5}, B = {1, 3, 5, 7} and C = {2, 4, 6, 8}, then verify that: A ∪ (B ∪ C) = (A ∪ B) ∪ C

Solution:

L.H.S = A ∪ (B ∪ C)
= {1, 2, 3, 4, 5} ∪ ({1, 3, 5, 7} ∪ {2, 4, 6, 8})
= {1, 2, 3, 4, 5} ∪ {l, 2, 3, 4, 5, 6, 7, 8}
= {1, 2, 3, 4, 5, 6, 7, 8}

R.H.S = (A ∪ B) ∪ C
= ({1, 2, 3, 4, 5} ∪ {1, 3, 5, 7}) ∪ {2, 4, 6, 8}
= {1, 2, 3, 4, 5, 7} ∪ {2, 4, 6, 8}
= {1, 2, 3, 4, 5, 6, 7, 8}

We see that L.H.S = R.H.S

- **Identity Property with respect to Union**

In sets, the empty set ϕ acts as identity for union, i.e. A ∪ ϕ = A

Example 3: If A = {*a, e, i, o, u*}, then verify that A ∪ ϕ = A.

Solution:

A ∪ ϕ = A
L.H.S = A ∪ ϕ

= {*a, e, i, o, u*} U { }
= {*a, e, i, o, u*} = A = R.H.S

Hence proved: L.H.S = R.H.S

- **Properties involving Intersection of Sets**

- **Commutative Property**

If A, B are any two sets, then

A ∩ B = B ∩ A

is called the commutative property of intersection of two sets.

Example 4: If a = {a, b, c, d} and B = {a, c, e, g}, then verify that A ∩ B = B ∩ A.

Solution:

A ∩ B = {a, b, c, d} ∩ {a, c, e, g}
= {a, c}
B ∩ A = {a, c, e, g} ∩ {a, b, c, d}
= {a, c}

From the above it is verified that A ∩ B = B ∩ A.

Example 5: If A= {1, 2, 3} and B = {4, 5, 6}, then verify that A ∩ B = B ∩ A.

Solution:

A ∩ B = {1, 2, 3} ∩ {4, 5, 6}
= { }
B ∩ A = {4, 5, 6} ∩ {1, 2, 3}
= { }

From the above it is verified that A ∩ B = B ∩ A.

- **Associative Property**

If A, B and C are any three sets, then A ∩ (B ∩ C) = (A ∩ B) ∩ C is called the associative property of intersection of three sets.

Example 6: If A = {1, 2, 5, 8}, B = {2, 4, 6} and C = {2, 4, 5, 7}, then verify that: A ∩ (B ∩ C) = (A ∩ B) ∩ C

Solution:

A = {1, 2, 5, 8}, B = {2, 4, 6}, C = {2, 4, 5, 7}

L.H.S = A ∩ (B ∩ C)
= {1, 2, 5, 8} ∩ ({2, 4, 6} ∩ {2, 4, 5, 7})
= {l, 2, 5, 8} ∩ {2, 4} = {2}

R.H.S = (A ∩ B) ∩ C
= ({l, 2, 5, 8} ∩ {2, 4, 6}) ∩ {2, 4, 5, 7}
= {2} ∩ {2, 4, 5, 7} = {2}

It is verified that L.H.S = R.H.S

- **Identity Property with respect to Intersection**

In sets, the universal set U acts as identity for intersection, i.e. A ∩ U = A.

Example 7: If U = {*a, b, c, ..., z*} and A= {*a, e, i, o, u*}, then verify that A ∩ U = A.

Solution:

U = {a, b, c, ...,z}, A= {a, e, i, o, u}

L.H.S = A ∩ U
= {a, e, i, o, u} ∩ {a, b, c, ..., z)
= {a, e, i, o, u}= A= R.H.S

Hence verified that L.H.S = R.H.S

- **Properties involving Difference of Sets**

If A and B are two unequal sets, then A – B ≠ B – A, For example if A = {0, 1, 2) and B = {1, 2, 3}, then

A – B = {0, 1, 2} – {1, 2, 3}
= {0}

B – A = {1, 2, 3} – {0, 1, 2}
= {3}

We can see that A – B ≠ B – A

- **Properties involving Complement of a Set**

Properties involving the sets and their complements are given below

A′ ∪ A = U   A ∩ A′ = ϕ   U′ = ϕ   ϕ′ = U

Example 8: If U = {1, 2, 3, ... ,10} and A= {1, 3, 5, 7, 9}, then prove that:

(i) U′ = ϕ **(ii)** A ∪ A′ = U **(iii)** A ∩ A′ = ϕ **(iv)** ϕ′ = U

Solution:

U = {1, 2, 3, ..., 10}, A= {1, 3, 5, 7, 9}

(i): U′ = ϕ

L.H.S = U′

We Know that U′ = U – U
= {1, 2, 3, ..., 10} - {1, 2, 3, ..., 10}
= { } = R.H.S

Hence verified that L.H.S = R.H.S

(ii): A U A′ = U

We know that A′ = U – A
= {1, 2, 3, ..., 10} - {1, 3, 5, 7, 9}
= {2, 4, 6, 8, 10}

Now we find,

A U A′ = {1, 3, 5, 7, 9} U {2, 4, 6, 8, 10}
= {1, 2, 3, ..., 10} = R.H.S

Hence verified that L.H.S = R.H.S

(iii) A ∩ A′ = ϕ

L.H.S = A ∩ A′
= {1, 3, 5, 7, 9} ∩ {2, 4, 6, 8, 10}
= { } = ϕ = R.H.S

Hence verified that L.H.S = R.H.S

(iv) ϕ′ = U

We know that

ϕ′ = U – ϕ
= {1, 2, 3, ... ,10} – { }
= {1, 2, 3, ... ,10} = U = R.H.S

Hence verified that L.H.S = R.H.S

EXERCISE 1.4

1. If A= {a, e, i, o, u}, B = {a, b, c} and C = {a, c, e, g}, then verify that:

(i) $A \cap B = B \cap A$ (ii) $A \cup B = B \cup A$ (iii) $B \cup C = C \cup B$
(iv) $B \cap C = C \cap B$ (v) $A \cap C = C \cap A$ (vi) $A \cup C = C \cup A$

2. If $X = \{1, 3, 7\}$, $Y = \{2, 3, 5\}$ and $Z = \{1, 4, 8\}$, then verify that:
(i) $X \cap (Y \cap Z) = (X \cap Y) \cap Z$ (ii) $X \cup (Y \cup Z) = (X \cup Y) \cup Z$

3. If $S = \{-2, -1, 0, 1\}$, $T = \{-4, -1, 1, 3\}$ and $U = \{0, \pm1, \pm2\}$, then verify that:
(i) $S \cap (T \cap U) = (S \cap T) \cap U$ (ii) $S u (T \cup U) = (S \cup T) \cup U$

4. If $O = \{1, 3, 5, 7.....\}$, $E = \{2, 4, 6, 8......\}$ and $N = \{1, 2, 3, 4....\}$, then verify that:
(i) $O \cap (E \cap N) = (O \cap E) \cap N$ (ii) $O \cup (E \cup N) = (O \cup E) \cup N$

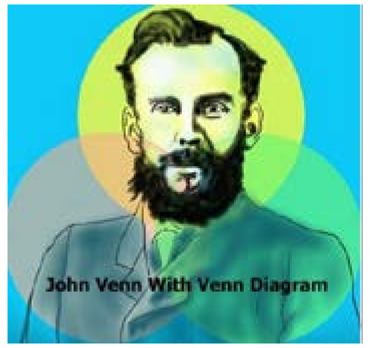
5. If $U = \{a, b, c, ...,z\}$, $S = \{a, e, i, o, u\}$ and $T = \{x, y, z\}$, then verify that:
(i) $S \cup \phi = S$ (ii) $T \cap U = T$ (iii) $S \cap S' = \phi$ (iv) $T \cup T' = U$

6. If $A = \{1, 7, 9, 11\}$, $B = \{1, 5, 9, 13\}$, and $C = \{2, 6, 9, 11\}$, then verify that:
(i) $A - B \neq B - A$ (ii) $A - C \neq C - A$

7. If $U = \{0, 1, 2,....,15\}$, $L = \{5, 7, 9,....,15\}$, and $M = \{6, 8, 10, 12, 14\}$, then verify the identity properties with respect to union and intersection of sets.

1.3 Venn Diagram

A Venn diagram is simple closed figures to show sets and the relationships between different sets.

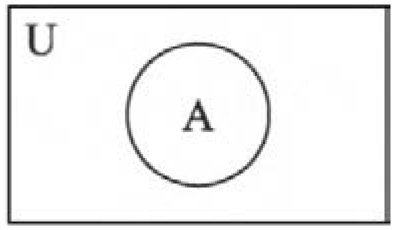


Venn diagram were introduced by a British logician and philosopher "John Venn" (1834 - 1923). John himself did not use the term "Venn diagram" Another logician "Lewis" used it first time in book "A survey of symbolic logic"

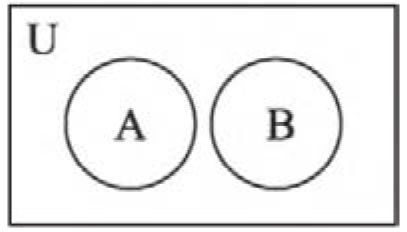
Animation 1.3: Venn diagram
Source & Credit: elearn.punjab

1.3.1 Representing Sets through Venn diagrams

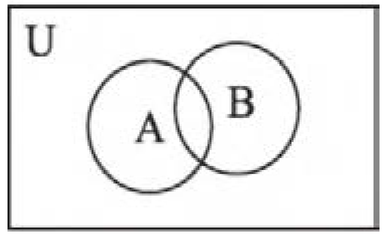
In Venn diagram, a universal set is represented by a rectangle and the other sets are represented by simple closed figures inside the rectangle. These closed figures show an overlapping region to describe the relationship between them. Following figures are the Venn diagrams for any set A of universal set U, disjoint sets A and B and overlapping sets A and B respectively.



Set A



Disjoint Sets



Overlapping Sets

In the Venn diagram, the shaded region is used to represent the result of operation.

1.3.2 Performing Operation on Sets through Venn Diagram

- **Union of Sets**

Now we represent the union of sets through Venn diagram when:

- **A is subset of B**

When all the elements of set A are also the elements of set B, then we can represent A U B by (figure i). Here shaded portion represents A U B.

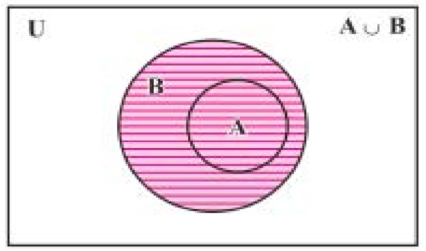


Figure (i)

- **B is subset of A**

When all the elements of set B are also the elements of set A, then we can represent A U B by (figure ii). Here shaded portion represents A U B.

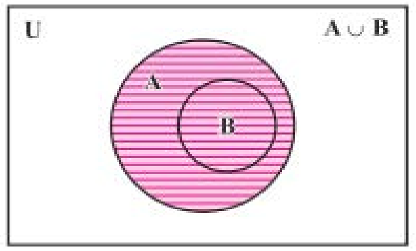


Figure (ii)

- **A and B are overlapping Sets**

When only a few elements of two sets A and B are common, then they are called overlapping sets. A U B is represented by (figure iii). Here shaded portion represents A U B.

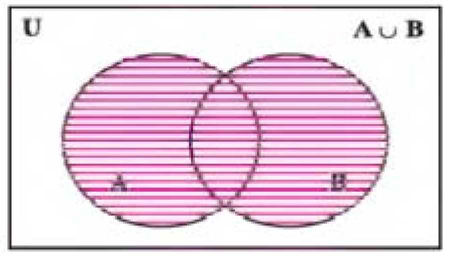


Figure (iii)

- **A and B are disjoint Sets**

When no element of two sets A and B is common, then we can represent A U B by (figure iv). Here shaded portion represents A U B.

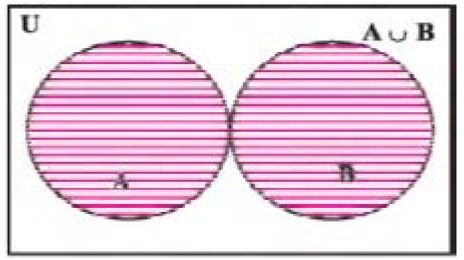


Figure (iv)

- **Intersection of Sets**

Now we clear the concept of intersection of two sets by using Venn diagram. In the given figures the shaded portion represents the intersection of two sets when:

- **A is subset of B**

When all the elements of set A are also the elements of set B, then we can represent A ∩ B by (figure v). Here shaded portion represents A ∩ B.

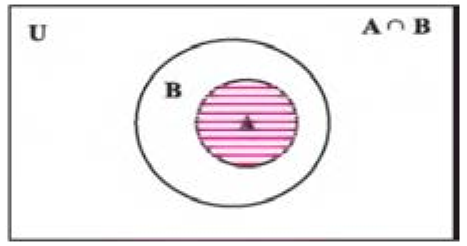


Figure (v)

- **B is subset of A**

When all the elements of set B are also the elements of set A, then we can represent $A \cap B$ by (figure vi). Here shaded portion represents $A \cap B$.

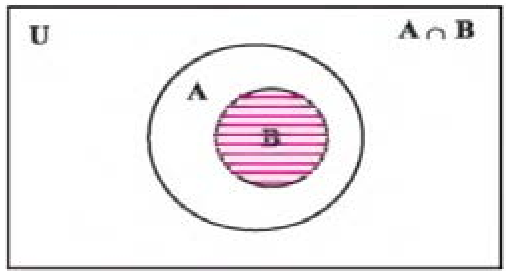


Figure (vi)

- **A and B are overlapping Sets**

When some elements are common, then we can represent $A \cap B$ by the fig (vii).

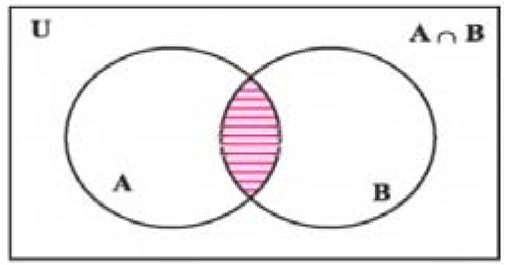


Figure (vii)

- **A and B are disjoint Sets**

When no element is common, then we can represent $A \cap B$ by fig (viii). So $A \cap B$ is an empty set.

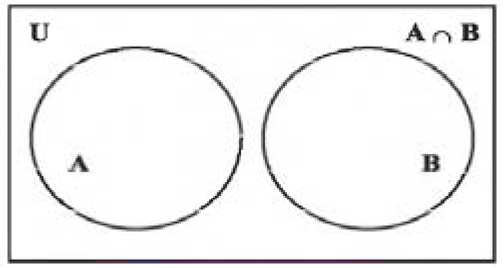


Figure (viii)

- **Difference of Two Sets A and B**

It is represented by shaded portion when:

- **A is subset of B**

When all the elements of set A are also the elements of set B, then we can represent $A - B$ by fig (ix).

There is no shaded portion. So, $A - B = \{\ \}$

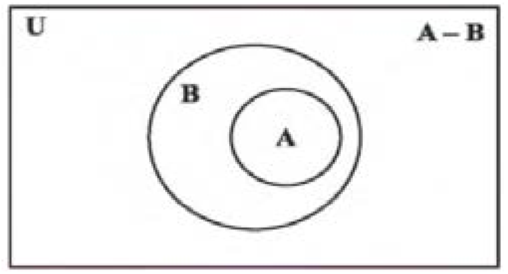


Figure (ix)

- **B is subset of A**

When all the elements of set B are also the elements of set A, then we can represent A - B by (figure x). Here shaded portion represents A - B.

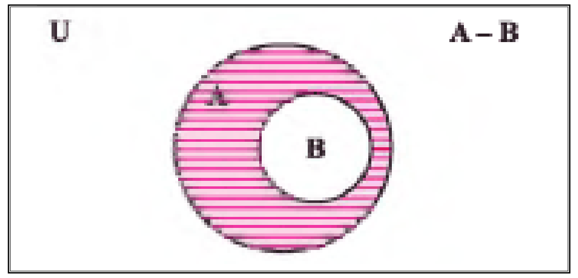


Figure (x)

- **A and B are overlapping Sets**

When some elements are common, then we can represent $A - B$ by the fig (xi).

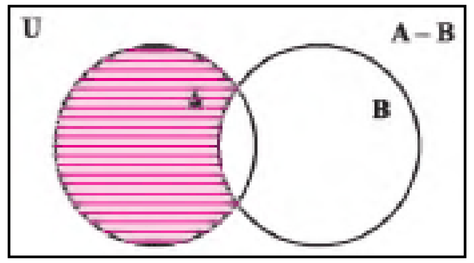


Figure (xi)

- **A and B are disjoint Sets**

When no element of two sets A and B is common, then we can represent A – B by (figure xii). Here shaded portion represents A – B.

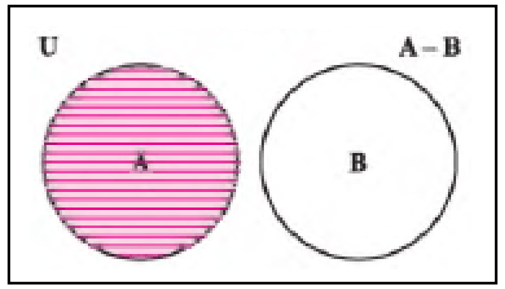
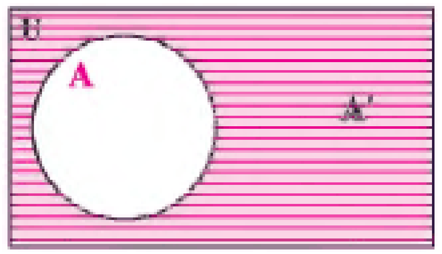


Figure (xii)

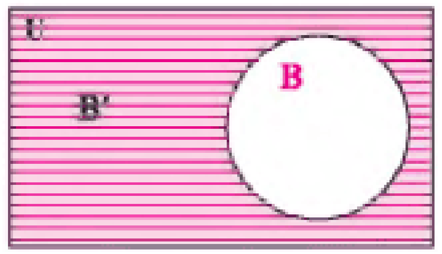
1.3.3 Complement of a Set

For complement of a set A



U – A = A′

For complement of a set B

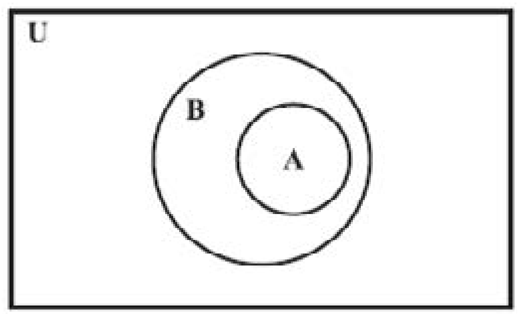


U – B = B′

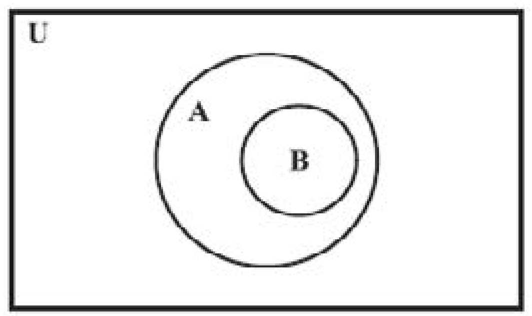
EXERCISE 1.5

1. Shade the diagrams according to the given operations.

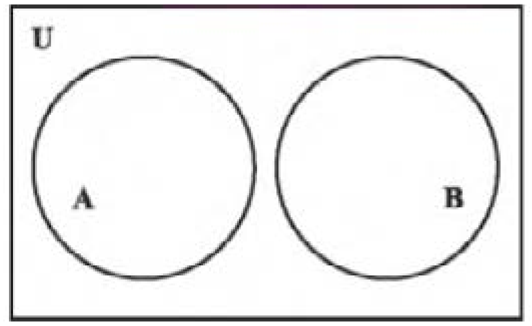
(i) A ∩ B (A is subset of B)



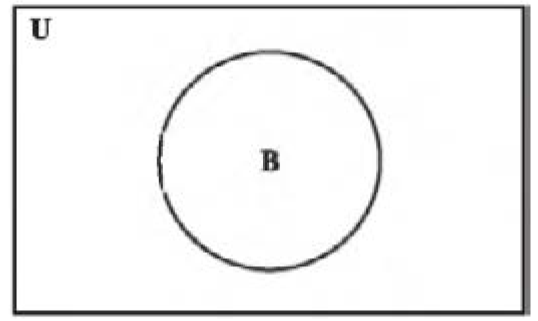
(ii) A ∪ B (B is subset of A)



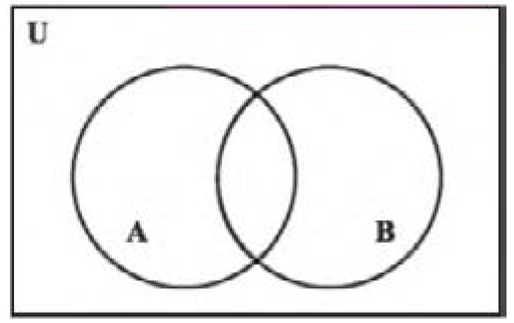
(iii) A – B (For disjoint sets)



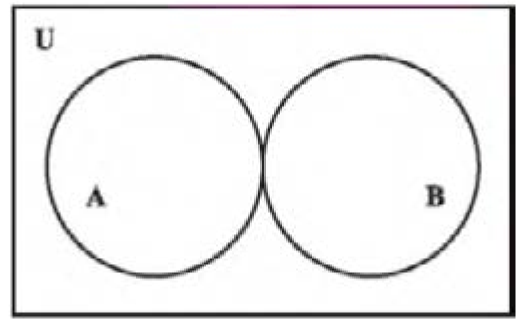
(iv) B′



(v) A ∩ B (Overlapping sets)



(vi) A ∪ B (For disjoint sets)



2. If, U= {1, 2, 3, 10}, A={1,4, 8, 9, 10} and B = {2, 3, 4, 7, 10}, then show that:
(i) A – B ≠ B – A (ii) A ∩ B = B ∩ A
(iii) A U B = B U A (iv) A′ ≠ B′
through Venn diagram.

Review Exercise 1

1. Answer the following questions.
(i) Name three forms for describing a set.
(ii) Define the descriptive form of set.
(iii) What does the symbol “ | ” mean ?
(iv) Write the name of the set consisting of all the elements of given sets under consideration.
(v) What is meant by disjoint sets?

2. Fill in the blanks.

(i) The symbol " ^ " means ______.

(ii) The set consisting of only common elements of two sets is called the ______ of two sets.

(iii) A set which contains all the possible elements of the sets under consideration is called the ______ set.

(iv) Two sets are called ______ if there is at least one element common between them and non of the sets is subset of the other.

(v) In sets, the universal set acts as ______ for intersection.

3. Tick (✓) the correct answer.

4. Write the following sets in the set builder form.

(i) A = {5, 6, 7, 8} (ii) B = {0, 1, 2}

(iii) C = {*a, e, i, o, u*}

(iv) D = set of natural numbers greater than 100

(v) E = set of odd numbers greater than 1 and less than 10

5. Write the following sets in descriptive and tabular form.

(i) $A = \{x \mid x \in W \wedge x < 7\}$

(ii) $B = \{x \mid x \in E \wedge 3 < x < 12\}$

(iii) $C = \{x \mid x \in Z \wedge -2 < x < +2\}$

(iv) $D = \{x \mid x \in P \wedge x < 15\}$

6. If A = {3, 4, 5, 6} and B = {2, 4, 6}, then verify that:

(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

7. If X = {2, 3, 4, 5} and Y = {1, 3, 5, 7}, then find:

(i) X – Y (ii) Y – X

8. If A = {*a, c, e, g*}, B = {*a, b, c, d*} and C = {*b, d, f, h*}, then verify that:

(i) $A \cup (B \cup C) = (A \cup B) \cup C$ (ii) $A \cap (B \cap C) = (A \cap B) \cap C$

9. If U = set of whole numbers and N = set of natural numbers, then verify that:

(i) $N' \cup N = U$ (ii) $N' \cap N = \phi$

10. If U = {*a, b, c, d, e*}, A = {*a, b, c*} and B = {*b, d, e*}, then show through Venn diagram

(i) A′ (ii) B′ (iii) A ∪ B (iv) A ∩ B

Summary

- There are three forms to write a set.
  (i) Descriptive form (ii) Tabular form (iii) Set builder form
- Two sets are said to be disjoint if there is no element common between them.
- If A and B are two sets then union of A and B is denoted by A ∪ B and intersection of A and B is denoted by A ∩ B.
- If A and B are two sets then B is said to be subset of A if every element of set B is the element of set A.
- Two sets are called overlapping sets if there is at least one element common between them but none of them is a subset of the other.
- A set which contains all possible elements of a given situation or discussion is called the universal set.